Sixth Grade Pacing Guide

Vision Statement

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.

National Council of Teachers of Mathematics

Process Standards

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-life data and situations within and outside mathematics and then apply appropriate strategies to find acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students become competent mathematical problem solvers.

Mathematical Communication

Students will use the language of mathematics, including specialized vocabulary and symbols, to express mathematical ideas precisely. Representing, discussing, reading, writing, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will learn to apply proportional and spatial reasoning and to reason from a variety of representations such as graphs, tables, and charts.

Mathematical Connections

Students will relate concepts and procedures from different topics in mathematics to one another and see mathematics as an integrated field of study. Through the application of content and process skills, students will make connections between different areas of mathematics and between mathematics and other disciplines, especially science. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships with a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should move easily among different representations—graphical, numerical, algebraic, verbal, and physical—and recognize that representation is both a process and a product.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.
- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain,* and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.
- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.
- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

6th Grade Math
Quarterly Overview Sheet

1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
Strand: Computation and Estimation **6.7** The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication and division of decimals. (C4) **Strand: Number and Number Sense** **6.5** The student will investigate and describe concepts of positive exponents and perfect squares(B2) . **Strand: Computation and Estimation** **6.8** ***The student will evaluate whole number numerical expressions, using the order of operations. (C3) **Strand: Patterns, Functions, and Algebra** **6.19** The student will investigate and recognize a) the identity properties of addition and multiplication (A1) b) the multiplicative property of zero (A1) **Strand: Number and Number Sense** **6.1** The student will describe and compare data, using ratios, and will use appropriate notations such as a/b, a to b, and a:b. (B2) **6.2**The student will a) investigate and describe fractions, decimals, and percents as ratios; (B2) b)identify a given fraction, decimal, or percent from a representation; (B1) c)*** demonstrate equivalent relationships among fractions, decimals, and percents; and (C3) d) ***compare and order fractions, decimals, and percents.(B2)	**Strand: Number and Number Sense** **6.4** The student will demonstrate multiple representations of multiplication and division of fractions. (C4) **Strand: Patterns, Functions, and Algebra** **6.19** The student will investigate and recognize c) the inverse property for multiplication. (A1) **Strand: Computation and Estimation** **6.6** The student will a) ***multiply and divide fractions and mixed numbers; and (C3) b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtractions, multiplication, and division of fractions. (C4) **Strand: Probability and Statistics** **6.14** The student, given a problem situation, will a) construct circle graphs; (C6) b) draw conclusions and make predictions, using circle graphs; and (D4) c) compare and contrast graphs that present information from the same data set. (C4) **6.15** The student will a) describe mean as a balance point; and (B2) b) decide which measure of center is appropriate for a given purpose. (A2)	**Strand: Probability and Statistics** **6.16** The student will a) compare and contrast dependent and independent events; and (B2) b) determine probabilities for dependent and independent events. (C4) **Strand: Number and Number Sense** **6.3** The student will a) identify and represent integers; (A1) b) order and compare integers; and (B2) c) identify and describe absolute value of integers. (B1) **Strand: Geometry** **6.11** The student will a) identify the coordinates of a point in a coordinate plane; and (A1) b) graph ordered pairs in a coordinate plane. (A3) **Strand: Patterns, Functions, and Algebra** **6.20** The student will graph inequalities on a number line. (C3) **6.17** The student will identify and extend geometric and arithmetic sequences. (B1 – C4) **Strand: Patterns, Functions, and Algebra** **6.18** The student will solve one-step linear equations in one variable involving whole number coefficients and positive rational solutions. (C3)	**Strand: Geometry** **6.12** The student will determine congruence of segments, angles, and polygons. (B4) **6.13** The student will describe and identify properties of quadrilaterals. (B2) **Strand: Measurement** **6.10** The student will a) define π (pi) as the ratio of the circumference of a circle to its diameter; (A1) b) solve practical problems involving circumference and area of a circle, given the diameter or radius; (C3) c) solve practical problems involving are and perimeter; and (C3) d) describe and determine the volume and surface area of a rectangular prism. (C3) **6.9** The student will make ballpark comparisons between measurements in the U. S. Customary System of Measurement and measurements in the metric system. (C3)

1st Quarter

Topic: Computation and Estimation

Focus: Applications of Operations with Rational Numbers

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard:6.7 The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication and division of decimals.

Understanding the Standard
Background information of teachers

- Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. For example: What is an approximate answer for 2.19 ÷ 0.8? The answer is around 2 because 2 ÷ 1 = 2.
- Understanding the placement of the decimal point is very important when finding quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, by 0.06, and by 0.006.
- Solving multistep problems in the context of real-life situations enhances interconnectedness and proficiency with estimation strategies.

  Examples of practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, deciding what time to leave for school or the movies, and sharing a pizza or the prize money from a contest.

Topic: Computation and Estimation

Focus: Applications of Operations with Rational Numbers

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.7 The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication and division of decimals.

Essential Knowledge and Skills	**Essential Questions** **Essential Understandings All Students should…**
Vocabulary: reasonableness, dividend, divisor, quotient, estimation, compatible numbers, front-end estimation, rounding	
• Solve single-step and multistep practical problems involving addition, subtraction, multiplication and division with decimals expressed to thousandths with no more than two operations.	What is the role of estimation in solving problems? Estimation gives a reasonable solution to a problem when an exact answer is not required. If an exact answer is required, estimation allows you to know if the calculated answer is reasonable.

Teacher Resources

VDOE Lesson Plans:

Practical Problems (VDOE)

Instructional Strategies:

- Budgets
- Shopping activities
- Help students see that there are various ways to estimate and that the choice of strategy depends on the task. To underscore that estimation is a problem-solving process, prompt students to consider first the goal of a task and the level of accuracy needed. Encourage students to invent their own estimation strategies. Encourage students to justify their estimation strategies.

Other Resources

Smart Lesson TEI

Smart Lesson TEI 2

Topic: Number and Number Sense

Focus: Relationships/Exponents and Perfect Squares

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.5 The student will investigate and describe concepts of positive exponents and perfect squares.

Understanding the Standard
Background information of teachers

- In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In 8^3, 8 is the base and 3 is the exponent.
- A power of a number represents repeated multiplication of the number by itself (e.g., $8^3 = 8 \times 8 \times 8$ and is read "8 to the third power").
- Any real number other than zero raised to the zero power is 1. Zero to the zero power (0) is undefined.
- Perfect squares are the numbers that result from multiplying any whole number by itself (e.g., $36 = 6 \times 6 = 6^2$).
- Perfect squares can be represented geometrically as the areas of squares the length of whose sides are whole numbers (e.g., 1×1, 2×2, or 3×3). This can be modeled with grid paper, tiles, geoboards and virtual manipulatives.

Topic: Number and Number Sense

Focus: Relationships/Exponents and Perfect Squares

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.5 The student will investigate and describe concepts of positive exponents and perfect squares.

Essential Knowledge and Skills **Vocabulary:** exponents, perfect squares, exponential notation, base, power of a number, zero power, undefined, powers of ten, factor	**Essential Questions** **Essential Understandings All Students should…**
• Recognize and describe patterns with exponents that are natural numbers, by using a calculator. • Recognize and describe patterns of perfect squares not to exceed 20^2, by using grid paper, square tiles, tables, and calculators. • Recognize powers of ten by examining patterns in a place value chart: $10^4 = 10{,}000$, $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$, $10^0 = 1$.	• What does exponential form represent? Exponential form is a short way to write repeated multiplication of a common factor such as $5 \times 5 \times 5 \times 5 = 5^4$. • What is the relationship between perfect squares and a geometric square? A perfect square is the area of a geometric square whose side length is a whole number.

Teacher Resources
VDOE Lesson Plans: Perfecting Squares (VDOE)
Instructional Strategies: unit squares -arrays on graph paper
Other Resources: Smart Lesson TEI

Topic:Computation and Estimation

Focus: Applications of Operations with Rational Numbers

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.8 The student will evaluate whole number numerical expressions, using the order of operations.

NOTE: 6.8 is taught without the use of a calculator.

Understanding the Standard
Background information of teachers

- The order of operations is a convention that defines the computation order to follow in simplifying an expression.
- The order of operations is as follows:
  - First, complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols, do the innermost operation first.
  - Second, evaluate all exponential expressions.
  - Third, multiply and/or divide in order from left to right.
  - Fourth, add and/or subtract in order from left to right.
- Parentheses ( ), brackets [ ], braces {}, and the division bar – as in $\frac{3+4}{5+6}$ should be treated as grouping symbols.
- The overuse of the acronym *PEMDAS* tends to reinforce inaccurate use of the order of operations. Students frequently multiply before dividing and add before subtracting because they do not understand the correct order of operations.
- The power of a number represents repeated multiplication of the number (e.g., $8^3 = 8 \cdot 8 \cdot 8$). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent.
- Any number, except 0, raised to the zero power is 1. Zero to the zero power is undefined.
- An *expression* is like a phrase. An expression has **no** equal sign and cannot be solved. Expressions are simplified by using the order of operations.

Example 1:
An expression can be simplified as follows:

$\frac{2+18}{9-4}\cdot(2^0+3)$

$\frac{2+18}{9-4}\cdot(1+3)$ Evaluate exponents first.

$\frac{20}{5}\cdot 4$ Evaluate within grouping symbols next.

$4\cdot 4$ Multiply.

16

Example 2:
Students also need to recognize which operation should be used first.

When simplifying the following, using order of operations, which operation should be performed first?

$8-4\div 2+3\cdot 5$

A $8-4$
B $4\div 2$
C $2+3$
D $3\cdot 5$

Example 3:
Sometimes the simplified value of an expression is in the form of an improper fraction.

The value of $\frac{2^2-1}{2}$ is $\frac{3}{2}$.

Topic: Computation and Estimation

Focus: Applications of Operations with Rational Numbers

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.8 The student will evaluate whole number numerical expressions, using the order of operations.

NOTE: 6.8 is taught without the use of a calculator.

Essential Knowledge and Skills	**Essential Questions** **Essential Understandings All Students should…**
Vocabulary: exponents, order of operations, expressions, base, grouping symbols, rational number	
• Simplify expressions by using the order of operations in a demonstrated step-by-step approach. The expressions should be limited to positive values and not include braces { } or absolute value | |. • Find the value of numerical expressions, using order of operations, mental mathematics, and appropriate tools. Exponents are limited to positive values.	• What is the significance of the order of operations? The order of operations prescribes the order to use to simplify expressions containing more than one operation. It ensures that there is only one correct answer. • What is the significance of the order of operations? The order of operations prescribes the order to use to simplify expressions containing more than one operation. It ensures that there is only one correct answer.

Teacher Resources

VDOE Lesson Plans:

Instructional Strategies:

- One player tosses the number cubes twice and all players record the numbers. The object of the game is to write an expression that uses all four numbers and all four operations to make a specified number, such as, the greatest number or the number closest to twenty-five. There is a time limit (e.g., two minutes). When time is called, players check each other's expressions. The player with the winning answer scores a point. After a few rounds, vary the rules (e.g., using parentheses or omitting multiplication).
- Write a long string of numbers and operations on the chalkboard. Give the students time to compute. Explore all the different answers to this single problem. Discuss the one correct answer and how the answer was obtained.
- Use a several different calculators to examine differences among calculators (e.g., answers). Students enter $6 + 2 \cdot 4 =$ on their calculators and compare each other's calculator displays. Some of the displays show 32 and others show 14. The students are asked: "Why", "Which is right?", "Are the other calculators broken?"
- Students work in pairs. They will select an index card containing an expression with a given value, missing the operations and/or grouping symbols. In order to arrive at the given value, the students will arrange the operations in correct order.

  For example:

  Expression: 6 7 5 12 6 2 Value is 38

  Answer: $6 + 7 \cdot 5 - 12 \div (6 - 2)$

Other Resources:

Topic:Patterns, Functions, and Algebra

Focus: Properties

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.19 The student will investigate and recognize
a) the identity properties of addition and multiplication
b) the multiplicative property of zero

Understanding the Standard
Background information of teachers

- Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division.
- The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., $5 + 0 = 5$).
- The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$).
- The multiplicative property of zero states that the product of any real number and zero is zero.
- Division by zero is not a possible arithmetic operation. Division by zero is undefined.

Topic:Patterns, Functions, and Algebra

Focus: Properties

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.19 The student will investigate and recognize
a) the identity properties of addition and multiplication
b) the multiplicative property of zero

Essential Knowledge and Skills	Essential Questions Essential Understandings All Students should…
Vocabulary: additive identity property, multiplicative identity property, multiplicative property of zero, real numbers, equations	
• Identify a real number equation that represents each property of operations with real numbers, when given several real number equations. • Test the validity of properties by using examples of the properties of operations on real numbers. • Identify the property of operations with real numbers that is illustrated by a real number equation. • NOTE: The commutative, associative and distributive properties are taught in previous grades.	• How are the identity properties for multiplication and addition the same? Different? For each operation the identity elements are numbers that combine with other numbers without changing the value of the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). • What is the result of multiplying any real number by zero? The product is always zero.

Teacher Resources
VDOE Lesson Plans: Pick and Choose (VDOE)
Instructional Strategies:
Other Resources: Smart Lesson TEI

Topic: Number and Number Sense

Focus: Relationships among Fractions, Decimals, and Percents

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.1 The student will describe and compare data, using ratios, and will use appropriate notations such as $\frac{a}{b}$, a to b, and a:b.

Understanding the Standard
Background information of teachers

- A ratio is a comparison of any two quantities. A ratio is used to represent relationships within and between sets.
- A ratio can compare part of a set to the entire set (part-whole comparison).
- A ratio can compare part of a set to another part of the same set (part-part comparison).
- A ratio can compare part of a set to a corresponding part of another set (part-part comparison).
- A ratio can compare all of a set to all of another set (whole-whole comparison).
- The order of the quantities in a ratio is directly related to the order of the quantities expressed in the relationship. For example, if asked for the ratio of the number of cats to dogs in a park, the ratio must be expressed as the number of cats to the number of dogs, in that order.
- A ratio is a multiplicative comparison of two numbers, measures, or quantities.
- All fractions are ratios and vice versa.
- Ratios may or may not be written in simplest form.
- Ratios can compare two parts of a whole.
- Rates can be expressed as ratios.

Topic: Number and Number Sense

Focus: Relationships among Fractions, Decimals, and Percents

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.1 The student will describe and compare data, using ratios, and will use appropriate notations such as $\frac{a}{b}$, a to b, and a:b.

Essential Knowledge and Skills **Vocabulary:** ratio, corresponding parts, comparison, relationships, notations, symbolic representation	**Essential Questions** **Essential Understandings All Students should…**
• Describe a relationship within a set by comparing part of the set to the entire set. • Describe a relationship between two sets by comparing part of one set to a corresponding part of the other set. • Describe a relationship between two sets by comparing all of one set to all of the other set. • Describe a relationship within a set by comparing one part of the set to another part of the same set. • Represent a relationship in words that makes a comparison by using the notations $\frac{a}{b}$, *a*:*b*, and *a to b*. • Create a relationship in words for a given ratio expressed symbolically.	• What is a ratio? A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a set and between two sets. A ratio can be written using fraction form ($\frac{2}{3}$), a colon (2:3), or the word *to* (2 to 3).

Teacher Resources
VDOE Lesson Plans: Field Goals, Balls, and Nets (VDOE)
Instructional Strategies:
Other Resources:

Topic: Number and Number Sense

Focus: Relationships among Fractions, Decimals, and Percents

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.2 The student will

a) investigate and describe fractions, decimals, and percents as ratios;
b) identify a given fraction, decimal, or percent from a representation;
c) demonstrate equivalent relationships among fractions, decimals, and percents; and
d) compare and order fractions, decimals, and percents.

NOTE: 6.2 c and d are taught without the use of a calculator

Understanding the Standard
Background information of teachers

- *Percent* means "per 100" or how many "out of 100"; *percent* is another name for *hundredths*.
- A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., 30% = $\frac{30}{100}$ = $\frac{3}{10}$ = 0.3).
- Percents can be expressed as fractions with a denominator of 100 (e.g., 75% = $\frac{75}{100}$ = $\frac{3}{4}$).
- Percents can be expressed as decimal (e.g., 38% = $\frac{38}{100}$ = 0.38).
- Some fractions can be rewritten as equivalent fractions with denominators of powers of 10, and can be represented as decimals or percents (e.g., $\frac{3}{5}$ = $\frac{6}{10}$ = $\frac{60}{100}$ = 0.60 = 60%).
- Decimals, fractions, and percents can be represented using concrete materials (e.g., Base-10 blocks, number lines, decimal squares, or grid paper).
- Percents can be represented by drawing shaded regions on grids or by finding a location on number lines.
- Percents are used in real life for taxes, sales, data description, and data comparison.
- Fractions, decimals and percents are equivalent forms representing a given number.
- The decimal point is a symbol that separates the whole number part from the fractional part of a number.
- The decimal point separates the whole number amount from the part of a number that is less than one.
- The symbol • can be used in Grade 6 in place of "x" to indicate multiplication.

- Strategies using 0, $\frac{1}{2}$ and 1 as benchmarks can be used to compare fractions.
- Continued…
- When comparing two fractions, use $\frac{1}{2}$ as a benchmark. Example: Which is greater, $\frac{4}{7}$ or $\frac{3}{9}$?

  $\frac{4}{7}$ is greater than $\frac{1}{2}$ because 4, the numerator, represents more than half of 7, the denominator. The denominator tells the number of parts that make the whole. $\frac{3}{9}$ is less than $\frac{1}{2}$ because 3, the numerator, is less than half of 9, the denominator, which tells the number of parts that make the whole. Therefore, $\frac{4}{7} > \frac{3}{9}$.
- When comparing two fractions close to 1, use distance from 1 as your benchmark. Example: Which is greater, $\frac{6}{7} or \frac{8}{9}$? $\frac{6}{7}$ is $\frac{1}{7}$ away from 1 whole. $\frac{8}{9} is \frac{1}{9}$ away from 1 whole. Since $\frac{1}{7} > \frac{1}{9}$, then $\frac{6}{7}$ is a greater distance away from 1 whole than $\frac{8}{9}$ so $\frac{8}{9} > \frac{6}{7}$.
- Students should have experience with fractions such as $\frac{1}{8}$, whose decimal representation is a terminating decimal (e. g., $\frac{1}{8}$ = 0.125) and with fractions such as $\frac{2}{9}$, whose decimal representation does not end but continues to repeat (e. g., $\frac{2}{9}$ = 0.222…). The repeating decimal can be written with ellipses (three dots) as in 0.222… or denoted with a bar above the digits that repeat as in $0.\overline{2}$.

Topic: Number and Number Sense

Focus: Relationships among Fractions, Decimals, and Percents

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.2 The student will

a) investigate and describe fractions, decimals, and percents as ratios;
b) identify a given fraction, decimal, or percent from a representation;
c) demonstrate equivalent relationships among fractions, decimals, and percents; and
d) compare and order fractions, decimals, and percents.

NOTE: 6.2 c and d are taught without the use of a calculator

Essential Knowledge and Skills **Vocabulary:** fraction, decimal, percent, ratio, representation, equvialent	**Essential Questions** **Essential Understandings All Students should…**
• Identify the decimal and percent equivalents for numbers written in fraction form including repeating decimals. • Represent fractions, decimals, and percents on a number line. • Describe orally and in writing the equivalent relationships among decimals, percents, and fractions that have denominators that are factors of 100. • Represent, by shading a grid, a fraction, decimal, and percent. • Represent in fraction, decimal, and percent form a given shaded region of a grid. • Compare two decimals through thousandths using manipulatives, pictorial representations, number lines, and symbols ($<, \leq, \geq, >, =$). • Compare two fractions with denominators of 12 or less using manipulatives, pictorial representations, number lines, and symbols ($<, \leq, \geq, >, =$). • Compare two percents using pictorial representations and symbols ($<, \leq, \geq, >, =$). • Order no more than 3 fractions, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less), in ascending or descending order.	What is the relationship among fractions, decimals and percents? Fractions, decimals, and percents are three different ways to express the same number. A ratio can be written using fraction form ($\frac{2}{3}$), a colon (2:3), or the word *to* (2 to 3). Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent.

Teacher Resources
VDOE Lesson Plans: Rational Speed Matching (VDOE)
Instructional Strategies:
Other Resources: Smart Lesson TEI 1 Smart Lesson TEI 2

2nd Quarter

Topic: Number and Number Sense

Focus: Fractions

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.4 The student will demonstrate multiple representations of multiplication and division of fractions.

Understanding the Standard
Background information of teachers

- Using manipulatives to build conceptual understanding and using pictures and sketches to link concrete examples to the symbolic enhance students' understanding of operations with fractions and help students connect the meaning of whole number computation to fraction computation.
- Multiplication and division of fractions can be represented with arrays, paper folding, repeated addition, repeated subtraction, fraction strips, pattern blocks and area models.
- When multiplying a whole by a fraction such as 3 x $\frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole.
- When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part.
- When multiplying a fraction by a whole number such as $\frac{1}{2}$ x 6, we are trying to find a part of the whole.
- For measurement division, the divisor is the number of groups. You want to know how many are in each of those groups. Division of fractions can be explained as how many of a given divisor is needed to equal the given dividend. In other words, for $\frac{1}{4} \div \frac{2}{3}$, the question is, "How many $\frac{2}{3}$ make $\frac{1}{4}$?"
- For partition division the divisor is the size of the group, so the quotient answers the question, "How much is the whole?" or "How much for one?"

Topic:Number and Number Sense

Focus: Fractions

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.4 The student will demonstrate multiple representations of multiplication and division of fractions.

Essential Knowledge and Skills **Vocabulary:** fractions, representation dividend, divisor, improper fraction, mixed number, product, proper fraction, quotient	**Essential Questions** **Essential Understandings All Students should…**
• Demonstrate multiplication and division of fractions using multiple representations. • Model algorithms for multiplying and dividing with fractions using appropriate representations.	• When multiplying fractions, what is the meaning of the operation? When multiplying a whole by a fraction such as 3 x $\frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part. When multiplying a fraction by a whole number such as $\frac{1}{2}$ x 6, we are trying to find a part of the whole. • What does it mean to divide with fractions? For measurement division, the divisor is the number of groups and the quotient will be the number of groups in the dividend. Division of fractions can be explained as how many of a given divisor are needed to equal the given dividend. In other words, for $\frac{1}{4} \div \frac{2}{3}$ the question is, "How many $\frac{2}{3}$ make $\frac{1}{4}$?" For partition division the divisor is the size of the group, so the quotient answers the question, "How much is the whole?" or "How much for one?"

Teacher Resources

VDOE Lesson Plans:

Modeling Division of Fractions (VDOE)

Modeling Multiplication of Fractions (VDOE)

Instructional Strategies:

- Find the LCD and LCM using prime factorization.
- Use the prime factorizations of numbers to multiply and divide fractions.

  For example $\frac{12}{5} \cdot \frac{25}{27} = \frac{2 \cdot 2 \cancel{3}}{\cancel{5}} \cdot \frac{5 \cdot \cancel{5}}{\cancel{3} \cdot 3 \cdot 3}$ (Common factors are divided to equal 1)

  Note: Remind students to replace common factors with 1 (not zero) to avoid error.
- Through teacher-orchestrated discussions of problems in context, students can develop useful methods to compute with fractions in ways that make sense. Students' understanding of computation can be enhanced by developing their own methods and sharing them with one another, explaining why their methods work and are reasonable to use, and then comparing their methods with the algorithms traditionally taught in school. In this way, students can appreciate the power and efficiency of the traditional algorithms and also connect them to student-invented methods that may sometimes be less powerful or efficient but are often easier to understand.
- Using an area model assists with students' developing understanding of multiplication and division of fractions.
- "Measurement and Fair-Sharing Models for Dividing Fractions", by Jeff Gregg and Diana Underwood Gregg, *Mathematics Teaching in the Middle School*, Vol. 12, No. 9, May, 2007, pages 490-496.

Other Resources:

Teaching Student Centered Mathematics, Grades 5-8, John Van de Walle and LouAnn Lovin, Pearson, 2006, pages 98-106. *Modeling strategies, various algorithms and activities for dividing fractions are presented.*

Topic:Patterns, Functions, and Algebra

Focus: Properties

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.19 The student will investigate and recognize
c) the inverse property for multiplication.

Understanding the Standard
Background information of teachers

- The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$).
- Zero has no multiplicative inverse.

Topic:Patterns, Functions, and Algebra

Focus: Properties

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard:6.19 The student will investigate and recognize

c) the inverse property for multiplication.

Essential Knowledge and Skills	Essential Questions Essential Understandings All Students should…
Vocabulary: real numbers, multiplicative inverse property additive identity property, associative property of addition , associative property of multiplication, commutative property of addition, commutative property of multiplication, distributive property identity elements, inverses, multiplicative identity property, multiplicative property of zero, reciprocal	
• Identify a real number equation that represents each property of operations with real numbers, when given several real number equations. • Test the validity of properties by using examples of the properties of operations on real numbers. • Identify the property of operations with real numbers that is illustrated by a real number equation. **NOTE: The commutative, associative and distributive properties are taught in previous grades.**	• Do all real numbers have a multiplicative inverse? No. Zero has no multiplicative inverse because there is no real number that can be multiplied by zero resulting in a product of one.

Teacher Resources
VDOE Lesson Plans: Pick and Choose (VDOE)
Instructional Strategies: • The class is divided into two teams of students. Each team is given 15 seconds to recognize the property of real numbers displayed on index cards. Each correct answer is worth one point. The team with the greatest number of points wins the game.
Other Resources:

Topic: Computation and Estimation

Focus: Application of Operations with Rational Numbers

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.6 The student will
a) multiply and divide fractions and mixed numbers; and
b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtractions, multiplication, and division of fractions.

NOTE: 6.6 a is taught without the use of a calculator

Understanding the Standard
Background information of teachers

- Simplifying fractions to simplest form assists with uniformity of answers.
- Addition and subtraction are inverse operations as are multiplication and division.
- It is helpful to use estimation to develop computational strategies. For example, $2\frac{7}{8} \cdot \frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3.
- When multiplying a whole by a fraction such as $3 \cdot \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole.
- When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part.
- When multiplying a fraction by a whole number such as $\frac{1}{2} \cdot 6$, we are trying to find a part of the whole.

Topic:**Computation and Estimation**

Focus: Application of Operations with Rational Numbers

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.6 The student will
a) multiply and divide fractions and mixed numbers; and
b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtractions, multiplication, and division of fractions.

NOTE: 6.6 a is taught without the use of a calculator

Essential Knowledge and Skills	**Essential Questions** **Essential Understandings All Students should…**
Vocabulary: fractions, mixed number, numerator, denominator, simplest form multiplicative inverse, estimation, reasonableness benchmark, compatible numbers, difference, improper fraction, product, proper fraction, quotient, reciprocal, simplest form, simplify, sum	
• Multiply and divide with fractions and mixed numbers. Answers are expressed in simplest form. • Solve single-step and multistep practical problems that involve addition and subtraction with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form. • Solve single-step and multistep practical problems that involve multiplication and division with fractions and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.	• How are multiplication and division of fractions and multiplication and division of whole numbers alike? Fraction computation can be approached in the same way as whole number computation, applying those concepts to fractional parts. • What is the role of estimation in solving problems? Estimation helps determine the reasonableness of answers.

Teacher Resources

VDOE Lesson Plans:

Modeling Multiplication of Fractions (VDOE)

Modeling Division of Fractions (VDOE)

Instructional Strategies:

- Find the LCD and LCM using prime factorization.
- Use the prime factorizations of numbers to multiply and divide fractions.

  For example $\frac{12}{5}\cdot\frac{25}{27}=\frac{2\cdot 2\cdot \cancel{3}}{\cancel{5}}\cdot\frac{\cancel{5}\cdot 5}{\cancel{3}\cdot 3\cdot 3}$
- NCTM Principals and Standards:
  In grades 6 – 8, students should acquire computational fluency—the ability to compute efficiently and accurately—with fractions, and decimals. Teachers should help students learn how to decide when an exact answer or an estimate would be more appropriate, how to choose the computational method that would be best to use, and how to evaluate the reasonableness of answers to computations. Most calculations should arise as students solve problems in context. Students should consider the features of the problem and the likely use of an answer to a calculation in deciding whether an exact answer or an estimate is needed, and then select an appropriate mode of calculation from among mental calculation, paper-and-pencil methods, or calculator use. For example, the cost of $1\frac{1}{4}$ pounds of cheese at $2.40 a pound can be found mentally, where as the cost of 1.37 pounds of cheese at $2.95 a pound might be estimated, although a calculator would probably be the preferred tool if an exact answer were needed. Students should remember to analyze the answers to their calculations to evaluate their reasonableness.
- Through teacher-orchestrated discussions of problems in context, students can develop useful methods to compute with fractions in ways that make sense. Students' understanding of computation can be enhanced by developing their own methods and sharing them with one another, explaining why their methods work and are reasonable to use, and then comparing their methods with the algorithms traditionally taught in school. In this way, students can appreciate the power and efficiency of the traditional algorithms and also connect them to student-invented methods that may sometimes be less powerful or efficient but are often easier to understand.

Other Resources:

Smart Lesson TEI a

Topic: Probability and Statistics

Focus: Practical Application of Statistics

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.14 The student, given a problem situation, will
a) construct circle graphs;
b) draw conclusions and make predictions, using circle graphs; and
c) compare and contrast graphs that present information from the same data set.

Understanding the Standard
Background information of teachers

- To collect data for any problem situation, an experiment can be designed, a survey can be conducted, or other data-gathering strategies can be used. The data can be organized, displayed, analyzed, and interpreted to answer the problem.
- Different types of graphs are used to display different types of data.
  - Bar graphs use categorical (discrete) data (e.g., months or eye color).
  - Line graphs use continuous data (e.g., temperature and time).
  - Circle graphs show a relationship of the parts to a whole.
- All graphs include a title, and data categories should have labels.
- A scale should be chosen that is appropriate for the data.
- A key is essential to explain how to read the graph.
- A title is essential to explain what the graph represents.
- Data are analyzed by describing the various features and elements of a graph.

Topic:Probability and Statistics

Focus: Practical Application of Statistics

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.14 The student, given a problem situation, will
a) construct circle graphs;
b) draw conclusions and make predictions, using circle graphs; and
c) compare and contrast graphs that present information from the same data set.

Essential Knowledge and Skills	Essential Questions Essential Understandings All Students should…
Vocabulary: circle graphs, bar graphs, categorical (discrete) data, line graphs, continuous data, title, data, categories, labels, scales, key, conclusions, predictions	
• Collect, organize and display data in circle graphs by depicting information as fractional. • Draw conclusions and make predictions about data presented in a circle graph. • Compare and contrast data presented in a circle graph with the same data represented in other graphical forms.	• What types of data are best presented in a circle graph? Circle graphs are best used for data showing a relationship of the parts to the whole.

Teacher Resources
VDOE Lesson Plans: May I Have Fries with That (VDOE)
Instructional Strategies:
Other Resources:

Topic: Probability and Statistics

Focus: Practical Application of Statistics

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.15 The student will
a) describe mean as a balance point; and
b) decide which measure of center is appropriate for a given purpose.

Understanding the Standard
Background information of teachers

- Measures of centerare types of averages for a data set. They represent numbers that describe a data set. Mean, median, and mode are measures of center that are useful for describing the average for different situations.
  - Mean works well for sets of data with no very high or low numbers.
  - Median is a good choice when data sets have a couple of values much higher or lower than most of the others.
  - Mode is a good descriptor to use when the set of data has some identical values or when data are not conducive to computation of other measures of central tendency, as when working with data in a yes or no survey.
- The mean is the numerical average of the data set and is found by adding the numbers in the data set together and dividing the sum by the number of data pieces in the set.
- In grade 5 mathematics, mean is defined as fair- share.
- Mean can be defined as the point on a number line where the data distribution is balanced. This means that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances of all the data points below the mean. This is the concept of mean as the balance point.
- Defining mean as balance point is a prerequisite for understanding standard deviation.
- The median is the middle value of a data set in ranked order. If there are an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values.
- The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there is more than one value that occurs most often, all these most-frequently-occurring values are modes. When there are exactly two modes, the data set is bimodal.

Topic:Probability and Statistics

Focus: Practical Application of Statistics

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.15 The student will
a) describe mean as a balance point; and
b) decide which measure of center is appropriate for a given purpose.

Essential Knowledge and Skills **Vocabulary:** measure of center, mean, balance point, median, mode, average	**Essential Questions** **Essential Understandings All Students should…**
• Find the mean for a set of data. • Describe the three measures of center and a situation in which each would best represent a set of data. • Identify and draw a number line that demonstrates the concept of mean as balance point for a set of data.	• What does the phrase "measure of center" mean? This is a collective term for the 3 types of averages for a set of data – mean, median, and mode. • What is meant by mean as balance point? Mean can be defined as the point on a number line where the data distribution is balanced. This means that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances of all the data points below the mean. This is the concept of mean as the balance point.

Teacher Resources
VDOE Lesson Plans: Balancing Act (VDOE)
Instructional Strategies:
Other Resources: Smart Notebook Mean as Balance Point

3rd Quarter

Topic: Probability and Statistics

Focus: Practical Application of Statistics

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.16 The student will
a) compare and contrast dependent and independent events; and
b) determine probabilities for dependent and independent events.

Understanding the Standard
Background information of teachers

- The probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes (sample space).
- The probability of an event occurring can be represented as a ratio or the equivalent fraction, decimal, or percent.
- The probability of an event occurring is a ratio between 0 and 1.
  - A probability of 0 means the event will never occur.
  - A probability of 1 means the event will always occur.
- A simple event is one event (e.g., pulling one sock out of a drawer and examining the probability of getting one color).
- Events are independent when the outcome of one has no effect on the outcome of the other. For example, rolling a number cube and flipping a coin are independent events.
- The probability of two independent events is found by using the following formula:

  $P(A \text{ and } B) = P(A) \cdot P(B)$

  Ex: When rolling two number cubes simultaneously, what is the probability of rollinga 3 on one cubeand a 4 on the other?

  $P(3 \text{ and } 4) = P(3) \cdot P(4) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

- Events are dependent when the outcome of one event is influenced by the outcome of the other. For example, when drawing two marbles from a bag, *not* replacing the first after it is drawn affects the outcome of the second draw.
- The probability of two dependent events is found by using the following formula:
  $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$

  Ex: You have a bag holding a blue ball, a red ball, and a yellow ball. What is the probability of picking a blue ball out of the bag on the first pick and then *without* replacing the blue ball in the bag, picking ared ball on the second pick?

$P(\text{blue and red}) = P(\text{blue}) \cdot P(\text{red after blue}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

Topic: Probability and Statistics

Focus: Practical Application of Statistics

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard:6.16 The student will
a) compare and contrast dependent and independent events; and
b) determine probabilities for dependent and independent events.

Essential Knowledge and Skills **Vocabulary:** probability of an event, simple event, dependent event, independent event, outcome	**Essential Questions** **Essential Understandings All Students should…**
• Determine whether two events are dependent or independent. • Compare and contrast dependent and independent events. • Determine the probability of two dependent events. • Determine the probability of two independent events.	• How can you determine if a situation involves dependent or independent events? Events are independent when the outcome of one has no effect on the outcome of the other. Events are dependent when the outcome of one event is influenced by the outcome of the other.

Teacher Resources
VDOE Lesson Plans: It Could Happen (VDOE)
Instructional Strategies:
Other Resources: Hershey's Kisses Activity (Harvey Almarode)

Topic:Number and Number Sense

Focus: Relationships among Integers

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.3 The student will
a) identify and represent integers;
b) order and compare integers; and
c) identify and describe absolute value of integers.

Understanding the Standard
Background information of teachers

- Integers are the set of whole numbers, their opposites, and zero.
- Positive integers are greater than zero.
- Negative integers are less than zero.
- Zero is an integer that is neither positive nor negative.
- A negative integer is always less than a positive integer.
- When comparing two negative integers, the negative integer that is closer to zero is greater.
- An integer and its opposite are the same distance from zero on a number line. For example, the opposite of 3 is -3.
- The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented as $|-6| = 6$.
- On a conventional number line, a smaller number is always located to the left of a larger number (e.g., –7 lies to the left of –3, thus –7 < –3; 5 lies to the left of 8 thus 5 is less than 8).

Topic: **Number and Number Sense**

Focus: Relationships among Integers

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.3 The student will
a) identify and represent integers;
b) order and compare integers; and
c) identify and describe absolute value of integers.

Essential Knowledge and Skills	**Essential Questions** **Essential Understandings All Students should…**
Vocabulary: integers, whole numbers, positive integers, negative integers, zero, absolute value, number line, opposites	
• Identify an integer represented by a point on a number line. • Represent integers on a number line. • Order and compare integers using a number line. • Compare integers, using mathematical symbols (<, >, =). • Identify and describe the absolute value of an integer.	• What role do negative integers play in practical situations? Some examples of the use of negative integers are found in temperature (below 0), finance (owing money), below sea level. There are many other examples. • How does the absolute value of an integer compare to the absolute value of its opposite? They are the same because an integer and its opposite are the same distance from zero on a number line.

Teacher Resources

VDOE Lesson Plans:

Ground Zero (VDOE)

Instructional Strategies:

number lines
-decks of cards
-Human number line

Other Resources

Smart Lesson TEI a

Smart lesson TEI b

How to create a number line in Word 2007
http://www.ehow.com/how_4963682_create-number-line-microsoft-word.html

Unit: **Patterns, Functions, and Algebra**

Topic: Geometry

Focus: Properties and Relationships/Coordinate Plane

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.11 The student will
a) identify the coordinates of a point in a coordinate plane; and
b) graph ordered pairs in a coordinate plane.

Understanding the Standard
Background information of teachers

- In a coordinate plane, the coordinates of a point are typically represented by the ordered pair (*x*, *y*), where *x* is the first coordinate and *y* is the second coordinate. However, any letters may be used to label the axes and the corresponding ordered pairs.
- The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular number lines. Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are (+,+); for quadrant II, (–,+); for quadrant III, (–, –); and for quadrant IV, (+,–).
- In a coordinate plane, the origin is the point at the intersection of the x-axis and y-axis; the coordinates of this point are (0,0).
- For all points on the x-axis, the y-coordinate is 0. For all points on the y-axis, the x-coordinate is 0.
- The coordinates may be used to name the point.(e.g., the point (2,7)). It is not necessary to say "the point whose coordinates are (2,7)".

Topic: Geometry

Focus: Properties and Relationships/Coordinate Plane

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.11 The student will
a) identify the coordinates of a point in a coordinate plane; and
b) graph ordered pairs in a coordinate plane.

Essential Knowledge and Skills	**Essential Questions** **Essential Understandings All Students should…**
Vocabulary: coordinate plane, coordinates, ordered pair, axes, quadrants coordinate, Origin, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate	
• Identify and label the axes of a coordinate plane. • Identify and label the quadrants of a coordinate plane. • Identify the quadrant or the axis on which a point is positioned by examining the coordinates (ordered pair) of the point. • Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. • Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. • Relate the coordinate of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line.	• Can any given point be represented by more than one ordered pair? The coordinates of a point define its unique location in a coordinate plane. Any given point is defined by only one ordered pair. • In naming a point in the plane, does the order of the two coordinates matter? Yes. The first coordinate tells the location of the point to the left or right of the y-axis and the second point tells the location of the point above or below the x-axis. Point (0, 0) is at the origin.

Teachers Resources

VDOE Lesson Plans

What's the Point (VDOE)

Instructional Strategies:

- Have students find and identify the missing coordinates of a vertex of a polygon on a coordinate plane.
- Using the board game, *Battleship*, students will learn the concept of ordered pairs as positions on a grid.
- Students will be given graph paper and ordered pairs will be called out orally. If the students have plotted the points correctly, a picture will be evident on the graph paper.
- Students will draw a picture on a coordinate graph and label their coordinates.
- Have students sit shoulder to shoulder one facing forward and the other facing backward. The first student will be given a picture on a coordinate plane. The other student will have a blank coordinate plane. The first student will describe the picture by calling out points on the coordinate plane and the second student will plot those points on their blank grid. When all the points have been given, students compare their coordinate planes to see if they are the same.

Other Resources:

Smart Lesson TEI

In a coordinate plane, the *origin* is the point at the intersection of the *x*-axis and *y*-axis; the coordinates of this point are (0, 0). For all points on the *x*-axis, the *y*-coordinate is 0 (e.g.. (2, 0), (–4, 0), (7, 0), and (–23, 0) are located on the *x*-axis.).For all points on the *y*-axis, the *x*-coordinate is 0. (e.g., (0, 2), (0, –7), (0, 14) and (0, –2) are located on the *y*-axis.). The coordinates may be used to name the point (e.g., the point (2, 7)). It is not necessary to say "the point whose coordinates are (2, 7)".

The coordinates may also be used to describe the distance (using absolute value) from both the *x*- and *y*-axis.

Example 1: The point $(3, -7)$ is 3 units from the *y*-axis and 7 units from the *x*-axis.

The coordinates may also be used to determine the distance from another point on the same horizontal or vertical line.

Example 2: Given two points on the same vertical line, (4, 8) and (4, 5), the distance between these points is the distance between 5 units and 8 units or 3 units.

Example 3: Given two points on the same vertical line, (2, 3) and $(2, -7)$, the distance between these points can be found by determining the distance each point is from the *x*-axis. (2, 3) is 3 units from the *x*-axis. $(2, -7)$ is 7 units from the *x*-axis. The distance from (2, 3) to $(2, -7)$ on the vertical line is 10 units.

Topic: Pattern, Functions, and Algebra

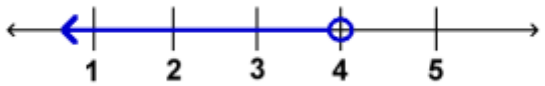
Focus: Inequalities

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.20 The student will graph inequalities on a number line.

Understanding the Standard
Background information of teachers

- Inequalities using the < or > symbols are represented on a number line with an open circle on the number and a shaded line over the solution set.
  Ex: x < 4



- When graphing x ≤ 4 fill in the circle above the 4 to indicate that the 4 is included.
- Inequalities using the ≤ *or* ≥ symbols are represented on a number line with a closed circle on the number and shaded line in the direction of the solution set.
- The solution set to an inequality is the set of all numbers that make the inequality true.
- It is important for students to see inequalities written with the variable before the inequality symbol and after. For example x > -6 and 7 > y.

Topic: Pattern, Functions, and Algebra

Focus: Inequalities

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.20 The student will graph inequalities on a number line.

Essential Knowledge and Skills **Vocabulary:** graph, inequality, symbols, solution set	**Essential Questions** **Essential Understandings All Students should…**
• Given a simple inequality with integers, graph the relationship on a number line. • Given the graph of a simple inequality with integers, represent the inequality two different ways using symbols (<, >, ≤, ≥).	• In an inequality, does the order of the elements matter? Yes, the order does matter. For example, $x > 5$ is not the same relationship as $5 > x$. However, $x > 5$ is the same relationship as $5 < x$.

Teacher Resources
VDOE Lesson Plans: Give or Take a Few (VDOE)
Instructional Strategies: Place copies of integer number lines in sheet protectors. Give each student a dry erase marker, eraser, and sheet protector with number line. Write an inequality on the board for students to see, and then have them graph the inequality on their number line.
Other Resources:

Topic: Pattern, Functions, and Algebra

Focus: Sequences

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.17 The student will identify and extend geometric and arithmetic sequences.

Understanding the Standard
Background information of teachers

- Numerical patterns may include linear and exponential growth, perfect squares, triangular and other polygonal numbers, or Fibonacci numbers.
- Arithmetic and geometric sequences are types of numerical patterns.
- In the numerical pattern of an arithmetic sequence, students must determine the difference, called the *common difference*, between each succeeding number in order to determine what is added to each previous number to obtain the next number. Sample numerical patterns are 6, 9, 12, 15, 18, …; and 5, 7, 9, 11, 13, ….
- In geometric number patterns, students must determine what each number is multiplied by to obtain the next number in the geometric sequence. This multiplier is called the *common ratio*. Sample geometric number patterns include 2, 4, 8, 16, 32, …; 1, 5, 25, 125, 625, …; and 80, 20, 5, 1.25, …
- Strategies to recognize and describe the differences between terms in numerical patterns include, but are not limited to, examining the change between consecutive terms, and finding common factors. An example is the pattern 1, 2, 4, 7, 11, 16,…

Topic: Pattern, Functions, and Algebra

Focus: Sequences

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.17 The student will identify and extend geometric and arithmetic sequences.

Essential Knowledge and Skills **Vocabulary:** numerical patterns, arithmetic sequences, common difference, geometric sequence, common ratio, consecutive terms, terms	**Essential Questions** **Essential Understandings All Students should…**
• Investigate and apply strategies to recognize and describe the change between terms in arithmetic patterns. • Investigate and apply strategies to recognize and describe geometric patterns. • Describe verbally and in writing the relationships between consecutive terms in an arithmetic or geometric sequence. • Extend and apply arithmetic and geometric sequences to similar situations. • Extend arithmetic and geometric sequences in a table by using a given rule or mathematical relationship. • Compare and contrast arithmetic and geometric sequences. • Identify the common difference for a given arithmetic sequence. • Identify the common ratio for a given geometric sequence.	• What is the difference between an arithmetic and a geometric sequence? While both are numerical patterns, arithmetic sequences are additive and geometric sequences are multiplicative.

Teacher Resources
VDOE Lesson Plans: Growing Patterns and Sequences (VDOE)
Instructional Strategies:
Other Resources:

Topic:Patterns, Functions, and Algebra

Focus: Variable Equations

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.18 The student will solve one-step linear equations in one variable involving whole number coefficients and positive rational solutions.

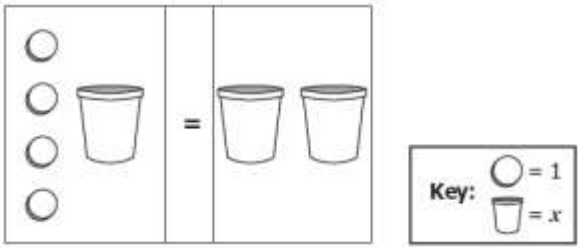
Understanding the Standard
Background information of teachers

- A one-step linear equation is an equation that requires one operation to solve.
- A mathematical expression contains a variable or a combination of variables, numbers, and/or operation symbols and represents a mathematical relationship. An expression cannot be solved.
- A term is a number, variable, product, or quotient in an expression of sums and/or differences. In $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and 3.
- A coefficient is the numerical factor in a term. For example, in the term $3xy^2$, 3 is the coefficient; in the term z, 1 is the coefficient.
- Positive rational solutions are limited to whole numbers and positive fractions and decimals.
- An equation is a mathematical sentence stating that two expressions are equal.
- A variable is a symbol (placeholder) used to represent an unspecified member of a set.

A variety of representations of equations should be used to demonstrate maintaining equality.

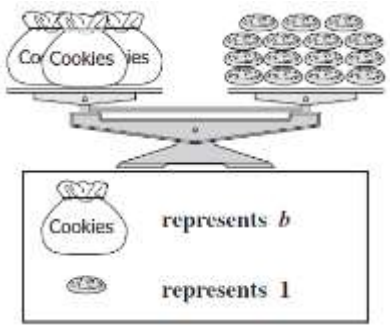
Example 1:
Example 2:

Look at the equation mat.

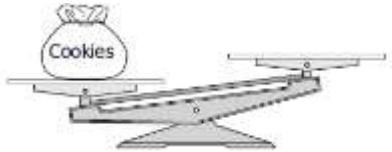


What is the value of x?

The scale below is balanced.



Using the above representations what could be placed on the right side of the following scale to make it balance?



Topic:Patterns, Functions, and Algebra

Focus: Variable Equations

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.18 The student will solve one-step linear equations in one variable involving whole number coefficients and positive rational solutions

Essential Knowledge and Skills	Essential Questions Essential Understandings All Students should…
Vocabulary: linear equations, term, variable, coefficient, equation, solution, expression numerical expression, variable expression	
• Represent and solve a one-step equation, using a variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale. • Solve a one-step equation by demonstrating the steps algebraically. • Identify and use the following algebraic terms appropriately: *equation*, v*ariable*, *expression*, *term*, and *coefficient*.	• When solving an equation, why is it necessary to perform the same operation on both sides of an equal sign? To maintain equality, an operation performed on one side of an equation must be performed on the other side.

Teacher Resources
VDOE Lesson Plans: Balanced (VDOE) Equation Vocabulary (VDOE)
Instructional Strategies: • Demonstrate the use of a pan balance (e.g., scale) by showing how equal numbers of like objects balance each other. • Play game of "Guess my Number" – or similar idea to demonstrate an unknown. "When you add 5 to me I become 12. Who Am I?" Use this to set the stage for unknowns. "How could we write this equation?"
Other Resources:

4th Quarter

Topic: Geometry

Focus: Properties and Relationships/Congruency

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.12 The student will determine congruence of segments, angles, and polygons.

Understanding the Standard
Background information of teachers

- Congruent figures have exactly the same size and the same shape.
- Noncongruent figures may have the same shape but not the same size.
- The symbol for congruency is $\cong$.
- The corresponding angles of congruent polygons have the same measure, and the corresponding sides of congruent polygons have the same measure.
- The determination of the congruence or noncongruence of two figures can be accomplished by placing one figure on top of the other or by comparing the measurements of all sides and angles.
- Construction of congruent line segments, angles, and polygons helps students understand congruency.

Topic: Geometry

Focus: Properties and Relationships/Congruency

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.12 The student will determine congruence of segments, angles, and polygons.

Essential Knowledge and Skills	**Essential Questions** **Essential Understandings All Students should…**
Vocabulary: congruent figures, noncongruent figures, coordinates, vertices, segments, angles, polygons, parallel sides, perpendicular, corresponding angles/sides, closed planar (two-dimensional) figure acute angle, hatch mark, line segment, obtuse angle, ray, right angle, straight angle	
• Characterize polygons as congruent and noncongruent according to the measures of their sides and angles. • Determine the congruence of segments, angles, and polygons given their attributes. • Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving practical and mathematical problems.	• Given two congruent figures, what inferences can be drawn about how the figures are related? The congruent figures will have exactly the same size and shape. • Given two congruent polygons, what inferences can be drawn about how the polygons are related? Corresponding angles of Congruent polygons will have the same measure. Corresponding sides of congruent polygons will have the same measure.

Teacher Resources

VDOE Lesson Plans:

Side to Side (VDOE)

Instructional Strategies:

- Angle legs
- geoboards
- Students look for examples of congruent figures within the environment.
- Play the "Congruent" Game. Cut out shapes of two each. Glue one of each shape on a circle with a spinner. Place all other pieces on another table. Divide class into two teams. Have a student come to the spinner and spin. Next, the student must go to the table and pick out the matching piece (e.g., visual memory), bring it back to the spinner, and see if it is a match. If the piece matches, the student's team receives a point. Continue rotating members and team turns until all students have had a turn.
- Use patty paper to compare figures to determine congruence or noncongruence.

Other Resources:

Topic: Geometry

Focus: Properties and Relationships

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.13 The student will describe and identify properties of quadrilaterals.

Understanding the Standard
Background information of teachers

- A quadrilateral is a closed planar (two-dimensional) figure with four sides that are line segments.
- A parallelogram is a quadrilateral whose opposite sides are parallel and opposite angles arecongruent.
- A rectangle is a parallelogram with four right angles.
- Rectangles have special characteristics (such as diagonals are bisectors) that are true for any rectangle.
- To bisect means to divide into two equal parts.
- A square is a rectangle with four congruent sides or a rhombus with four right angles.
- A rhombus is a parallelogram with four congruent sides.
- A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases*, and the nonparallel sides are called *legs*. If the legs have the same length, then the trapezoid is an isosceles trapezoid.
- A kite is a quadrilateral with two pairs of adjacent congruent sides. One pair of opposite angles is congruent.
- Quadrilaterals can be sorted according to common attributes, using a variety of materials.
- Quadrilaterals can be classified by the number of parallel sides: a parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has only one pair of parallel sides; other quadrilaterals have no parallel sides.
- Quadrilaterals can be classified by the measures of their angles: a rectangle has four 90° angles; a trapezoid may have zeroor two 90° angles.
- Quadrilaterals can be classified by the number of congruent sides: a rhombus has four congruent sides; a square, which is a rhombus with four right angles, also has four congruent sides; a parallelogram and a rectangle each have two pairs of congruent sides.
- A square is a special type of both a rectangle and a rhombus, which are special types of parallelograms, which are special types of quadrilaterals.
- The sum of the measures of the angles of a quadrilateral is 360°.

- A chart, graphic organizer, or Venn Diagram can be made to organize quadrilaterals according to attributes such as sides and/or angles.

Topic: Geometry

Focus: Properties and Relationships

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.13 The student will describe and identify properties of quadrilaterals.

Essential Knowledge and Skills	Essential Questions Essential Understandings All Students should…
Vocabulary: polygon, quadrilateral, parallelograms, rectangles, trapezoids, kites, rhombi, squares, congruent sides, parallel sides, angles, Venn Diagram, bisect, congruent, diagonal, intersect, isosceles trapezoid, parallel, perpendicular, perpendicular bisector, quadrilateral, rectangle	
• Sort and classify polygons as quadrilaterals, parallelograms, rectangles, trapezoids, kites, rhombi, and squares based on their properties. Properties include number of parallel sides, angle measures and number of congruent sides. • Identify the sum of the measures of the angles of a quadrilateral as 360°.	• Can a figure belong to more than one subset of quadrilaterals? Any figure that has the attributes of more than one subset of quadrilaterals can belong to more than one subset. For example, rectangles have opposite sides of equal length. Squares have all 4 sides of equal length thereby meeting the attributes of both subsets.

Teacher Resources

VDOE Lesson Plans:

Exploring Quadrilaterals (VDOE)

Instructional Strategies:

- Venn diagrams
- geoboards
- graphic organizer or foldable

- Given a set of different quadrilaterals, students will classify them according to common attributes, which are determined by the students. Following a discussion by the students about their classifications, the teacher will refine their definitions.
- Cooperative Activity: Each group is given a large loop of yarn. Without talking each student in the group must have both hands on the yarn and must make a shape specified by the teacher.

Other Resources:

Topic: Measurement

Focus: Problems Solving with Area, Perimeter, Volume, and Surface Area

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.10 The student will

a) define π (pi) as the ratio of the circumference of a circle to its diameter;
b) solve practical problems involving circumference and area of a circle, given the diameter or radius;
c) solve practical problems involving area and perimeter; and
d) describe and determine the volume and surface area of a rectangular prism.

Understanding the Standard
Background information of teachers

- Experiences in deriving the formulas for area, perimeter, and volume using manipulatives such as tiles, one-inch cubes, adding machine tape, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and facility in their use.
- The perimeter of a polygon is the measure of the distance around the polygon.
- Circumference is the distance around or perimeter of a circle.
- The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve.
- The perimeter of a square whose side measures s is 4 times s ($P = 4s$), and its area is side times side ($A = s^2$).
- The perimeter of a rectangle is the sum of twice the length and twice the width [$P = 2l + 2w$, or $P = 2(l + w)$], and its area is the product of the length and the width ($A = lw$).
- The value of pi (π) is the ratio of the circumference of a circle to its diameter.
- The ratio of the circumference to the diameter of a circle is a constant value, pi (π), which can be approximated by measuring various sizes of circles.
- The fractional approximation of pi generally used is $\frac{22}{7}$.
- The decimal approximation of pi generally used is 3.14.
- The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle.
- The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle.

- The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$).
- The volume of a rectangular prism is computed by multiplying the area of the base, B, (length x width) by the height of the prism ($V = lwh = Bh$).

Topic: Measurement

Focus: Problems Solving with Area, Perimeter, Volume, and Surface Area

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.10 The student will

a) define π (pi) as the ratio of the circumference of a circle to its diameter;
b) solve practical problems involving circumference and area of a circle, given the diameter or radius;
c) solve practical problems involving are and perimeter; and
d) describe and determine the volume and surface area of a rectangular prism.

Essential Knowledge and Skills	Essential Questions Essential Understandings All Students should…
Vocabulary: circumference, circle, radius, diameter, area, perimeter, volume, surface area, rectangular prism, pi, faces, bases, height, three-dimensional figure, two-dimensional figure, linear units, cubic units, square units, chord, cube, formula, height, length,, net, parallelogram, ratio, rectangle, side, square, trapezoid, triangle, width	
• Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. • Find the circumference of a circle by substituting a value for the diameter or the radius into the formula $C = \pi d$ or $C = 2\pi r$. • Find the area of a circle by using the formula $A = \pi r^2$. • Apply formulas to solve practical problems involving area and perimeter of triangles and rectangles. • Create and solve problems that involve finding the circumference and area of a circle when given the diameter or radius. • Solve problems that require finding the surface area of a rectangular prism, given a diagram of the prism with the necessary dimensions labeled. • Solve problems that require finding the volume of a rectangular prism given a diagram of the prism with the necessary dimensions labeled.	• What is the relationship between the circumference and diameter of a circle? The circumference of a circle is about 3 times the measure of the diameter. • What is the difference between area and perimeter? Perimeter is the distance around the outside of a figure while area is the measure of the amount of space enclosed by the perimeter. • What is the relationship between area and surface area? Surface area is calculated for a three-dimensional figure. It is the sum of the areas of the two-dimensional surfaces that make up the three-dimensional figure.

Teacher Resources

VDOE Lesson Plans:

Going the Distance (VDOE)

Out of the Box (VDOE)

Instructional Strategies:

- Formula foldable
- Dog pen activity
- Volume activity with packing peanuts
- Give the students a circular object (e.g., plastic lids, cups, etc.), two different colors of strings, and a centimeter ruler. Students work in groups of two. The pair will measure the distance around the object and the diameter. Regardless of the lid size used, if the students have been careful wrapping the string, they should get three diameters out of the string length with a little string left over. After all partners have finished their measurements and calculations for the circles, write the ratios (e.g., circumference/diameter), including repeats, that they found in decimal form on the chalkboard. Ask the students to find the average (e.g., mean) of all ratios found. When students have measured carefully and 20 or more ratios are averaged together, the mean is usually quite close to 3.14.
- Ask students to write a sentence that describes a diameter's relationship to its circumference (e.g., a circumference equals about 3.14 of the diameter).
- Draw a rectangle, 7 units long and 5 units wide. Find the perimeter. Draw as many rectangles that you can find with perimeters of 24 units.
- Use geoboards or dot paper to explore area and perimeter.
- Demonstrate a number of different polygons with a given area and/or perimeter.
- Students bring in cereal and oatmeal boxes from home and cut them apart to more easily determine the surface area.
- Students stack unit cubes in various ways and find the surface areas of the structures they have built. They sketch their figures and discuss which figure has the largest surface area and which has the smallest surface area.
- Create models with concrete objects: string, ribbon, strips of paper to measure perimeter, and develop formulas.
- Use squares of paper, tiles, cardboard, and carpet to develop the concept of area.
- Use geoboards to model figures and to determine area and perimeter.
- Using pattern blocks, construct two different size parallelograms, triangles, and trapezoids. Next, measure their bases and heights. Using pencil and paper or calculator, find the area of each shape and compare the answers.
- Using wooden cubes and grid paper, have students draw a 3x4 rectangle on the paper and place 12 cubes on the original layer. Have students arrive at the formula for the volume of a rectangular prism by multiplying the area of the base (number of cubes in each layer) times the number of layers (height).

Other Resources:

Looking at Geometry, Grades 6-9, Aims Education Foundation, 2009, pages 31-54. *Activities are designed to develop and use formulas for area of rectangles, parallelograms, triangles, and trapezoids.*

Teaching Student Centered Mathematics, Grades 5-8, John Van de Walle and LouAnn Lovin, Pearson Education, 2006, Pages 251–256. *Activities are designed to develop and use formulas for area of rectangles, parallelograms, triangles, and trapezoids.*

Topic: Measurement

Focus: Problem Solving/U.S. Customary and Metric Systems

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.9 The student will make ballpark comparisons between measurements in the U. S. Customary System of Measurement and measurements in the metric system.

Understanding the Standard
Background information of teachers

- Making sense of various units of measure is an essential life skill, requiring reasonable estimates of what measurements mean, particularly in relation to other units of measure.
  - 1 inch is about 2.5 centimeters.
  - 1 foot is about 30 centimeters.
  - 1 meter is a little longer than a yard, or about 40 inches.
  - 1 mile is slightly farther than 1.5 kilometers.
  - 1 kilometer is slightly farther than half a mile.
  - 1 ounce is about 28 grams.
  - 1 nickel has the mass of about 5 grams.
  - 1 kilogram is a little more than 2 pounds.
  - 1 quart is a little less than 1 liter.
  - 1 liter is a little more than 1 quart.
  - Water freezes at 0°C and 32°F.
  - Water boils at 100°C and 212°F.
  - Normal body temperature is about 37°C and 98°F.
  - Room temperature is about 20°C and 70°F.
- Mass is the amount of matter in an object. Weightis the pull of gravity on the mass of an object.The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight*, as shown by the questions: "How much does it weigh?" versus "What is its mass?"
- The degree of accuracy of measurement required is determined by the situation.
- Whether to use an underestimate or an overestimate is determined by the situation.
- Physically measuring objects along with using visual and symbolic representations improves student understanding of both the concepts and processes of measurement.

Topic: Measurement

Focus: Problem Solving/U.S. Customary and Metric Systems

Process standards: The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations.

Standard: 6.9 The student will make ballpark comparisons between measurements in the U. S. Customary System of Measurement and measurements in the metric system.

Essential Knowledge and Skills	**Essential Questions** **Essential Understandings All Students should…**
Vocabulary: U. S. Customary System, inch, foot (feet), yard, mile, ounce, pounds, metric system, centimeters, meter, kilometer, weight, mass, grams, kilograms, freezes, boils	
• Estimate the conversion of units of length, weight/mass, volume, and temperature between the U.S. Customary system and the metric system by using ballpark comparisons. Ex: 1 L ≈ 1qt. Ex: 4L ≈ 4 qts. • Estimate measurements by comparing the object to be measured against a benchmark.	• What is the difference between weight and mass? Weight and mass are different. Mass is the amount of matter in an object. Weight is the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. • How do you determine which units to use at different times? Units of measure are determined by the attributes of the object being measured. Measures of length are expressed in linear units, measures of area are expressed in square units, and measures of volume are expressed in cubic units. • Why are there two different measurement systems? Measurement systems are conventions invented by different cultures to meet their needs. The U.S. Customary System is the preferred method in the United States. The metric system is the preferred system worldwide.

Teacher Resources
VDOE Lesson Plans: Measuring Mania (VDOE)
Instructional Strategies: Metric Me Activity
Other Resources: