Probability and Statistics (not AP and not Dual Enrolled) Pacing Guide

Vision Statement

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it. National Council of Teachers of Mathematics

1st Quarter

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(First 4½ Weeks)</td><td>PS.8 The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.</td><td>• The value of a sample statistic varies from sample to sample if the simple random samples are taken repeatedly from the population of interest.

• Poor data collection can lead to misleading and meaningless conclusions.</td><td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

• Compare and contrast controlled experiments and observational studies and the conclusions one can draw from each.

• Compare and contrast population and sample and parameter and statistic.

• Identify biased sampling methods.

• Describe simple random sampling.

• Select a data collection method appropriate for a given context.</td></tr>
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Time Frame	**SOL Objective/ Competency**	**Essential Understandings/Questions**	**Essential Knowledge/Skills**
1st Quarter (First 4½ Weeks)	PS.9 The student will plan and conduct a survey. The plan will address sampling techniques (e.g., simple random, stratified) and methods to reduce bias.	• The purpose of sampling is to provide sufficient information so that population characteristics may be inferred. • Inherent bias diminishes as sample size increases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. • Determine which sampling technique is best, given a particular context. • Plan a survey to answer a question or address an issue. • Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias. • Design a survey instrument. • Conduct a survey.

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
1st Quarter (First 4½ Weeks)	PS.10 The student will plan and conduct an experiment. The plan will address control, randomization, and measurement of experimental error.	• Experiments must be carefully designed in order to detect a cause-and-effect relationship between variables. • Principles of experimental design include comparison with a control group, randomization, and blindness.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Plan and conduct an experiment. The experimental design should address control, randomization, and minimization of experimental error.

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
1st Quarter (First 4½ Weeks)	PS.1 The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.	• Data are collected for a purpose and have meaning in a context. • Measures of central tendency describe how the data cluster or group. • Measures of dispersion describe how the data spread (disperse) around the center of the data. • Graphical displays of data may be analyzed informally. • Data analysis must take place within the context of the problem.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Create and interpret graphical displays of data, including dotplots, stem-and-leaf plots, and histograms. • Examine graphs of data for clusters and gaps, and relate those phenomena to the data in context. • Examine graphs of data for outliers, and explain the outlier(s) within the context of the data. • Examine graphs of data and identify the central tendency of the data as well as the spread. Explain the central tendency and the spread of the data within the context of the data.

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
1st Quarter (First 4½ Weeks)	PS.2 The student will analyze numerical characteristics of univariate data sets to describe patterns and departures from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers.	• Data are collected for a purpose and have meaning within a context. • Analysis of the descriptive statistical information generated by a univariate data set should include the interplay between central tendency and dispersion as well as among specific measures. • Data points identified algorithmically as outliers should not be excluded from the data unless sufficient evidence exists to show them to be in error.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem's context. • Identify possible outliers, using an algorithm. • Explain the influence of outliers on a univariate data set. • Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
1st Quarter (First 4½ Weeks)	PS.3 The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.	• Data are collected for a purpose and have meaning in a context. • Statistical tendency refers to typical cases but not necessarily to individual cases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast two or more univariate data sets by analyzing measures of center and spread within a contextual framework. • Describe any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data. • Analyze in context kurtosis and skewness in conjunction with other descriptive measures.

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
1st Quarter (First 4½ Weeks)	PS.4 The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points.	• A scatterplot serves two purposes: – to determine if there is a useful relationship between two variables, and – to determine the family of equations that describes the relationship. • Data are collected for a purpose and have meaning in a context. • Association between two variables considers both the direction and strength of the association. • The strength of an association between two variables reflects how accurately the value of one variable can be predicted based on the value of the other variable. • Outliers are observations with large residuals and do not follow the pattern apparent in the other data points.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Examine scatterplots of data, and describe skewness, kurtosis, and correlation within the context of the data. • Describe and explain any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data. • Identify influential data points (observations that have great effect on a line of best fit because of extreme *x*-values) and describe the effect of the influential points.

2nd Quarter

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
2nd Quarter (Second 4½ Weeks)	PS.7 The student, using two-way tables, will analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.	• Data are collected for a purpose and have meaning in a context. • Simpson's paradox refers to the fact that aggregate proportions can reverse the direction of the relationship seen in the individual parts. • Two categorical variables are independent if the conditional frequencies of one variable are the same for every category of the other variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Produce a two-way table as a summary of the information obtained from two categorical variables. • Calculate marginal, relative, and conditional frequencies in a two-way table. • Use marginal, relative, and conditional frequencies to analyze data in two-way tables within the context of the data.

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
2nd Quarter (Second 4½ Weeks)	PS.15 The student will identify random variables as independent or dependent and find the mean and standard deviations for sums and differences of independent random variables.	• A random variable is a variable that has a single numerical value, determined by chance, for each outcome of a procedure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast independent and dependent random variables. • Find the standard deviation for sums and differences of independent random variables.

Time Frame	**SOL Objective/ Competency**	**Essential Understandings/Questions**	**Essential Knowledge/Skills**
2nd Quarter (Second 4½ Weeks)	PS.11 The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.	• The complement of event A consists of all outcomes in which event A does not occur. • Two events, A and B, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If A and B are not independent, then they are said to be dependent. • Events A and B are mutually exclusive if they cannot occur simultaneously.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. • Given two or more events in a problem setting, determine if the events are complementary, dependent, independent, and/or mutually exclusive.

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
2nd Quarter (Second 4½ Weeks)	PS.12 The student will find probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the Law of Large Numbers concept, the addition rule, and the multiplication rule.	• Data are collected for a purpose and have meaning in a context. • Venn diagrams may be used to find conditional probabilities. • The Law of Large Numbers states that as a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Calculate relative frequency and expected frequency. • Find conditional probabilities for dependent, independent, and mutually exclusive events.

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
2nd Quarter (Second 4½ Weeks)	PS.14 The student will simulate probability distributions, including binomial and geometric.	• A probability distribution combines descriptive methods and probabilities to form a theoretical model of behavior. • A probability distribution gives the probability for each value of the random variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design and conduct an experiment that simulates a binomial distribution. • Design and conduct an experiment that simulates a geometric distribution.

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
2^{nd} Quarter (Second 4½ Weeks)	PS.13 The student will develop, interpret, and apply the binomial probability distribution for discrete random variables, including computing the mean and standard deviation for the binomial variable.	• A probability distribution is a complete listing of all possible outcomes of an experiment together with their probabilities. The procedure has a fixed number of independent trials. • A random variable assumes different values depending on the event outcome. • A probability distribution combines descriptive statistical techniques and probabilities to form a theoretical model of behavior.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Develop the binomial probability distribution within a real-world context. • Calculate the mean and standard deviation for the binomial variable. • Use the binomial distribution to calculate probabilities associated with experiments for which there are only two possible outcomes.

3rd Quarter

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
3rd Quarter (Third 4½ Weeks)	PS.16 The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities, using a table or graphing calculator.	• The normal distribution curve is a family of symmetrical curves defined by the mean and the standard deviation. • Areas under the curve represent probabilities associated with continuous distributions. • The normal curve is a probability distribution and the total area under the curve is 1.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the properties of a normal probability distribution. • Describe how the standard deviation and the mean affect the graph of the normal distribution. • Determine the probability of a given event, using the normal distribution.

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
3rd Quarter (Third 4½ Weeks)	PS.19 The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.	• The Central Limit Theorem states: – The mean of the sampling distribution of means is equal to the population mean. – If the sample size is sufficiently large, the sampling distribution approximates the normal probability distribution. – If the population is normally distributed, the sampling distribution is normal regardless of sample size. • Sampling distributions have less variability with larger sample sizes.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the use of the Central Limit Theorem for drawing inferences about a population parameter based on a sample statistic. • Describe the effect of sample size on the sampling distribution and on related probabilities. • Use the normal approximation to calculate probabilities of sample statistics falling within a given interval. • Identify and describe the characteristics of a sampling distribution of a sample proportion, mean, difference between two sample proportions, or difference between two sample means.

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
3rd Quarter (Third 4½ Weeks)	PS.17 The student, given data from a large sample, will find and interpret point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, and difference between two means (independent and paired).	• A primary goal of sampling is to estimate the value of a parameter based on a statistic. • Confidence intervals use the sample statistic to construct an interval of values that one can be reasonably certain contains the true (unknown) parameter. • Confidence intervals and tests of significance are complementary procedures. • Paired comparisons experimental design allows control for possible effects of extraneous variables.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct confidence intervals to estimate a population parameter, such as a proportion or the difference between two proportions; or a mean or the difference between two means. • Select a value for alpha (Type I error) for a confidence interval. • Interpret confidence intervals in the context of the data. • Explain the importance of random sampling for confidence intervals. • Calculate point estimates for parameters and discuss the limitations of point estimates.

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3rd Quarter (Third 4½ Weeks)	PS.20 The student will identify properties of a t-distribution and apply t-distributions to single-sample and two-sample (independent and matched pairs) t-procedures, using tables or graphing calculators.	• Paired comparisons experimental design allows control for possible effects of extraneous variables. • The sampling distribution of means with a small sample size follows a t-distribution.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the properties of a t-distribution. • Compare and contrast a t-distribution and a normal distribution. • Use a t-test for single-sample and two-sample data.

4th Quarter

Time Frame	SOL Objective/ Competency	Essential Understandings/Questions	Essential Knowledge/Skills
4th Quarter (Last 4½ Weeks)	PS.18 The student will apply and interpret the logic of a hypothesis-testing procedure. Tests will include large sample tests for proportion, mean, difference between two proportions, and difference between two means (independent and paired) and Chi-squared tests for goodness of fit, homogeneity of proportions, and independence.	• Confidence intervals and tests of significance are complementary procedures. • Paired comparisons experimental design allows control for possible effects of extraneous variables. • Tests of significance assess the extent to which sample data support a hypothesis about a population parameter. • The purpose of a goodness of fit test is to decide if the sample results are consistent with results that would have been obtained if a random sample had been selected from a population with a known distribution. • Practical significance and statistical significance are not necessarily congruent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use the Chi-squared test for goodness of fit to decide if the population being analyzed fits a particular distribution pattern. • Use hypothesis-testing procedures to determine whether or not to reject the null hypothesis. The null hypothesis may address proportion, mean, difference between two proportions or two means, goodness of fit, homogeneity of proportions, and independence. • Compare and contrast Type I and Type II errors. • Explain how and why the hypothesis-testing procedure allows one to reach a statistical decision.

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4th Quarter (Last 4½ Weeks)	PS.3 The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.	• Data are collected for a purpose and have meaning in a context. • Statistical tendency refers to typical cases but not necessarily to individual cases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast two or more univariate data sets by analyzing measures of center and spread within a contextual framework. • Describe any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data. • Analyze in context kurtosis and skewness in conjunction with other descriptive measures.

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4th Quarter (Last 4½ Weeks)	PS.5 The student will find and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity.	• Data are collected for a purpose and have meaning in a context. • Least squares regression generates the equation of the line that minimizes the sum of the squared distances from the data points to the line. • Each data point may be considered to be comprised of two parts: fit (the part explained by the model) and residual (the result of chance variation or of variables not measured). • Residual = Actual – Fitted • A correlation coefficient measures the degree of association between two variables that are related linearly. • Two variables may be strongly associated without a cause-and-effect relationship existing between them.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Calculate a correlation coefficient. • Explain how the correlation coefficient, *r*, measures association by looking at its formula. • Use regression lines to make predictions, and identify the limitations of the predictions. • Use residual plots to determine if a linear model is satisfactory for describing the relationship between two variables. • Describe the errors inherent in extrapolation beyond the range of the data. • Use least squares regression to find the equation of the line of best fit for a set of data. • Explain how least squares regression generates the equation of the line of best fit by examining the formulas used in computation.

Time Frame	**SOL Objective/ Competency**	**Essential Understandings/Questions**	**Essential Knowledge/Skills**
4th Quarter (Last 4½ Weeks)	PS.6 The student will make logarithmic and power transformations to achieve linearity.	• A logarithmic transformation reduces positive skewness because it compresses the upper tail of the distribution while stretching the lower tail. • Nonlinear transformations do not preserve relative spacing between data points.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Apply a logarithmic transformation to data. • Explain how a logarithmic transformation works to achieve a linear relationship between variables. • Apply a power transformation to data. • Explain how a power transformation works to achieve a linear relationship between variables.